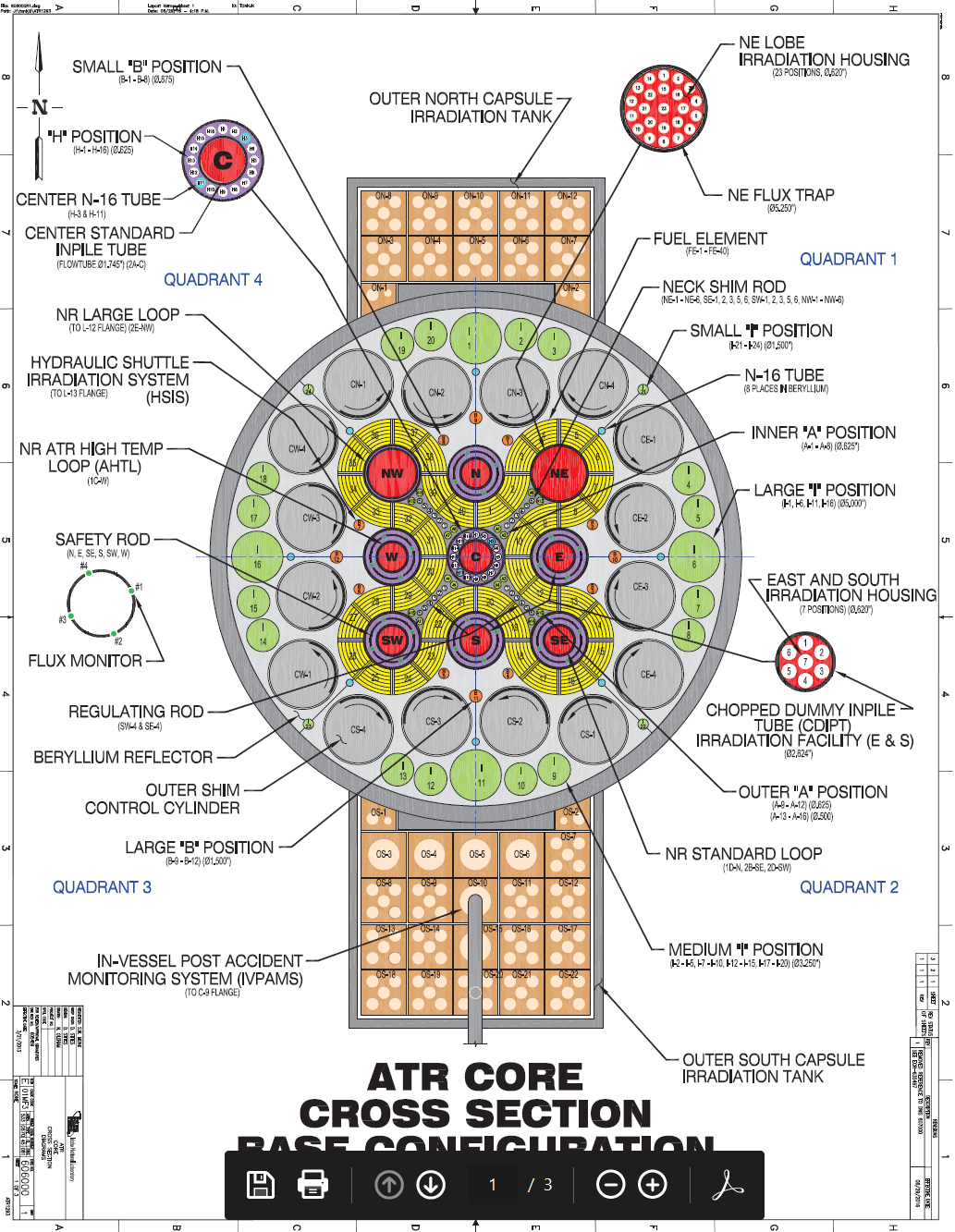
# Abstract

This project began as a way to enhance core physics analysis efficiencies and reduce modeling and analysis time. To that end, thumb rules and techniques previously used by ATR reactor engineering were scrutinized and adjusted to ensure optimal efficiency. The reactivity worth of U-235 and B-10 were calculated using HELIOS to help reactor engineering personnel efficiently select a mixture of new and recycled fuel elements to ensure that the requested cycle length could be achieved and that the fuel would not exceed any established limits. Using the results of the reactivity calculation, a new algorithm was developed to efficiently select initial guesses for outer shim control cylinder position for input into HELIOS to perform cycle depletion calculations. This algorithm greatly reduces the time required to perform a depletion analysis. The Tappendorf method of determining reactivity change was validated using HELIOS. Additional restrictions on the correlation were observed and will need to be incorporated into the standard methodology. Perturbation theory was applied to fuel elements with various gram loading in two different ways and then compared to the HELIOS calculated eigen value. This results of these comparisons show that the reactivity change from a small perturbation whether experiment worth change or fuel element change can be calculated without having to re-perform any HELIOS calculations.

# Introduction

The Advanced Test Reactor (ATR) is a 250 MW pressurized water reactor operated by the Department of Energy. This reactor is designed for the express purpose of performing materials testing for structural and fuel samples for both commercial and naval applications. Because this reactor is a test reactor, each operating cycle is unique and must be analyzed prior to operation. These cycles are typically 60 days long, operate nominally between 100 MW and 200 MW and have an approximately 30 day refueling outage between cycles.

One of the unique features of ATR is that the power in each lobe (NW, NE, C, SW, SE) can be independently controlled and maintained by the Outer Shim Control Cylinders (OSCCs). Thus, researchers can specify power requirements in the four ordinal directions (center power is a function of these directions), to custom tailor the neutron flux to their experiments. Another parameter that can be specified is the cycle length. This specification of parameters makes each cycle challenging to design and the short time between cycles enhances the difficulty of cycle modeling to demonstrate core safety.

From the design and history of the ATR, many thumb rules and correlations have been developed to help shorten the time it takes to model the cycle and develop the cycle physics analysis (CPA). As is usually the case with thumb-rules, they begin to be generalized to applications for which they were not designed. Over time the origins of these thumb-rules and correlations are lost to time and need to be re-proven. This re-proving was done in this work, specifically the worth of U-235 and B-10. This work also validates the Tappendorf correlation and states the applicability of the correlation as applied to various components within the reactor core. Beyond that, additional algorithms were developed to help shorten the CPA process, specifically to shorten the time it takes to perform the depletion calculations and to use first-order perturbation theory to predict the change on critical eigen value for small changes in reactivity such that the entire cycle depletion does not have to be re-performed.

# Reactivity Determination

## Background

Each Advanced Test Reactor cycle is governed by a Reactor Loading Record (RLR) which has requests from different research organizations for the nominal lobe power plus and minus a tolerance value and an operating cycle length. The requirements can only be satisfied if the correct amount of fuel is selected for that cycle. ATR fuel is comprised of 40 individual fuel elements arranged in a serpentine or 4-leaf clover pattern. Each of the quadrants have Outer Shim Control Cylinders which are beryllium cylinders that have a hafnium arc on a 120° section. The beryllium serves as a reflector while the hafnium is a neutron absorber. These OSCCs have two functions, the first of which is to compensate for fuel burnup. As fuel is depleted in the core throughout the cycle, the OSCC are rotated outward, meaning the hafnium is less exposed to the core. This removes poison from the system allowing neutrons to be reflected into the core. These neutrons can cause fission, so in effect less fuel is needed to get the same fission rate. The second purpose of the OSCC is to “shape” the flux in the reactor. Each quadrant power can be independently controlled, within reason, to accommodate the request of different experiment sponsors. To ensure that the cycle length can be met with the power tilt that is requested, each quadrant needs to be loaded differently. Quadrants with a high power request need to have more fuel loaded to ensure that power request can be met without the OSCC being so far withdrawn that there is no ability to compensate for fuel burn up.

Traditionally, ATR Reactor Engineering has relied on rules of thumb to help with loading each quadrant correctly. These rules of thumb were expressed as grams of fuel per dollar of reactivity. One of the main purposes of this project was to validate this thumb rule and/or propose a new one. This is to ensure that fuel loading can be optimized. Too little fuel will result in a shortened cycle and too much fuel can result in unacceptable power peaking and decreased fuel economy.

## Test Setup

The fuel reactivity worth was determined computationally using the 2D neutronics code HELIOS. HELIOS is a multigroup deterministic neutron transport solver that is currently used by ATR Reactor Engineering to perform the core physics analysis (CPA) prior to each operating cycle. It has a proven track record in ATR core performance. The simulation was initially set up by selecting 40 fresh fuel elements. These elements have the maximum amount of uranium for the fuel element. By using all fresh elements, any effect of fission product poison or other physical unknown was negated. The simulation was run using an eigen value calculator within HELIOS. This calculator determines eigen values for 3.75°, 48.75°, 82.50°, 116.25°, and finally at 153.75°. Once these eigen values were determined and set as a baseline against which to benchmark, the first series of testing began.

## Test 1

The first test was performed by selecting a fuel element that had 254 grams less fuel than a fresh fuel element. This element was placed in the model at position 1. The five eigen value calculation was reperformed and documented. This process was repeated 39 more times, once for each fuel element position in the fuel annulus. The eigen values were collected and tabulated. Each eigen value calculation took about 1.5 hours for a total of 60 hours of high-performance computing time for the test. Several shell scripts were written to help automate the process, so the time requirement between element runs wasn’t especially onerous.

## Test 2

Test 2 was performed the same way as test 1. An element that had exactly 300 grams less U-235 than a fresh fuel element was selected and modeled in each position in the fuel annulus.

## Test 3

Test 3 was performed using a fuel element that had 100 grams less U-235 than a fresh fuel element. The idea behind this test was to see if there was a significant difference in reactivity worth calculation depending on fuel loading, and to see if a linear trend could be established that could be used by ATR Reactor Engineering to help select fuel.

## Test 4

Test 4 was completed using a fuel element that had 300 grams less U-235 than a fresh fuel element like test 2. This element however was a “non-borated” element which means that in the solver, there were no effects of boron included. This test served two purposes; first to determine the worth of boron in a fuel element, and second to eliminate an uncertainty in test 2 by removing the effect of depleted boron. By comparing the eigen value calculations from this test to those of Test 2 a reactivity worth for boron could be quantified.

## Reactivity Test Results

The reactivity for each test was calculated by comparing the eigen value at 3.75° shim position. This was done with the following equation:

Equation 1

Where:

is the 3.75° eigen value from the test case

is the 3.75° eigen value from the control case

β is the delayed neutron fraction taken to be 0.0075 for the Advanced Test Reactor

By dividing the equation by β, the reactivity is reported in terms of dollars which is a measure of distance to prompt critical. For example, if a system were to have 1$ of reactivity added during critical operations, the reactor would be exactly prompt critical.

Figure 1 shows the reactivity change of the element with 254 grams less U-235 than a fresh element.

Figure : Reactivity worth for element with 254-gram delta

Similar trends can be seen in figures 2,3, and 4 which show the delta 300 g, 100 g, and 300 g non-borated elements respectively.

Figure : Reactivity worth for element with 300-gram delta

Figure : Reactivity worth for element with 100-gram delta

Figure : Reactivity worth for element with 300-gram delta non-borated

In each test, the same general trend is observed. The reactivity worth changes the most for the centrally located elements while elements on the periphery have smaller reactivity changes. This is expected as reactivity is a function of local neutron flux. The neutron flux generally is higher in the center of the core than on the periphery which is what explains the shape of figures 1-4.

The next step was to compare each test against one another to see if the relative magnitudes are equivalent. This is done in figure 5 where the results of each test are plotted on the same figure.

Figure : Reactivity Comparison

In figure 5, it is seen that the same trends exist for each test case. As expected, the elements that are closer in gram loading to fresh elements have the least reactivity change while the elements that are farther away from fresh in terms of gram loading have the largest reactivity differences. As could be expected, the non-borated case shows the largest reactivity difference. As the boron burns out in a standard fuel element, reactivity increases. What was not expected was the miniscule difference between the Delta 254 case and the Delta 300 case. Perhaps the Delta 254 case could be though of a bounding case, meaning that any more decrease in U-235 content won’t translate to a larger reactivity difference because the surrounding fresh fuel dominate the reaction at that point. In essence, it can be assumed that once a fuel element has lost 250 grams of fissile material, it can be counted as not even there. Perhaps a better case would be the Delta 300 NB for a bounding element. Additional testing would need to be done to explore this limit and establish if a limit does indeed exist in this context. Additionally, the limit may be artificial; an artifact of the way the test was constructed and for a normal cycle with a mix of fresh and recycled elements, this limit might not exist.

To determine a “thumb-rule” number for reactivity worth for U-235, several ideas were pursued. As seen in the previous figures, the reactivity worth is strongly dependent on position. The question, then, is should thumb-rules be proposed that are position dependent or should it be an average of the core? Should the 100-gram case be considered to be representative or should the 300 gram case be used? Because ATR generally uses a mix of fresh and recycled fuel elements, a strategy that combined the results of the borated elements was used. Non-borated element use is less frequent, so the results of that test were not included in the final determination. The results for each test were averaged for elements in the respective lobe, NW, NE, C, SW, and SE. This resulted in a gram/dollar worth of 4109, 2775, 871, 2531, and 2159 g/$ respectively. These lobe averages were then averaged into a single value that is hopefully representative of the entire core. **Thus, the new thumb-rule for reactivity worth of fuel is 2489 g U-235/$ reactivity.**

The determination of Boron worth is a little trickier. From figure 5 the non-borated case has a slightly larger reactivity change than the borated 300-gram delta case. The only difference in the two cases is the inclusion of the boron which in this case was 0.1 grams. Thus for 0.1 grams, the reactivity difference can be calculated by subtracting the reactivity difference of the 300-gram borated case with the 300 gram non-borated case. The average between the two cases is approximately 0.02$. **Therefore, the worth of boron is about 0.21$ per gram or 4.86 grams per dollar of reactivity.**

# Fuel Depletion Algorithm

## Background

Each cycle in the Advanced Test Reactor is modeled using HELIOS to determine if the planned operation will be enveloped by the current safety analysis. As such, the cycle is modeled from beginning to end including several depletion steps in the middle. The depletions steps are modeled to ensure that 1) there is enough reactivity at each step to maintain the requested power level, and 2) to ensure that burnup limits are not exceeded. Both of these parameters feed in to the overall safety analysis for the cycle.

A standard 60-day cycle is generally divided into about 10 day time steps, with a few additional smaller time steps at the beginning of the proposed operation to provide extra detail during startup and the first few days until the reactor has fully come to equilibrium. At each time step, the user is required to input the position of the Outer Shim Control Cylinders and the Neck Shim Rods to ensure that the requested power in the lobe can be maintained and that the reactivity in the core doesn’t deviate significantly from critical. In practice, the analyst attempts to maintain plus or minus 1$ from critical if not better. It is important to note that 1$ away from critical doesn’t mean that the reactor is prompt critical, but rather that it would be difficult to maintain a given power level with that much extra reactivity. A prompt critical condition results from the insertion of 1$ of positive reactivity which is different than what is being reported by HELIOS.

Once the positions of the various shims are determined for each time step, the depletion calculation is performed using INL’s high performance computing cluster. For each time step, two eigen values are computed, one at the beginning of the step and one at the end of the step. This is required to allow HELIOS to employ a predictor-corrector algorithm to provide more accurate results. Each eigen value calculation can take as little as 30 minutes, but as many as 60 minutes. The average is usually about 45 minutes. Thus for 20 eigen values, the calculation takes approximately 15 hours. Once the depletion is completed, the analyst reviews the data and makes changes to shim positions based on lobe power and overall reactivity deviation from the critical condition. This process is mostly guess and check. Until now, there has not been a consistently utilized or accurate method to determine what the shim positions are at each step. Most analysts guess the shim positions for the first time step and perform the depletion before guessing the second time step shim positions. Unfortunately, the entire depletion calculation has to be run for each time step every time. The program isn’t currently capable of reading in the fuel and other component compositions at the end of the previously used time step, but rather can only read in the beginning of cycle compositions. This process then takes several days to complete, depending primarily on the experience of the analyst. For a person performing the depletion calculation for the first time, this process may take up to four weeks. A seasoned user may be able to complete the process in about two weeks.

## Development

The goal in developing a fuel depletion algorithm was to reduce the time required to perform the depletion calculation. The idea was that by knowing how much fuel was burned during each time step and knowing what the reactivity worth of that fuel is, the outer shim control cylinders could be withdrawn to compensate for the fuel depletion. In this way, a more educated guess for the OSCC positions could be made for HELIOS input at each time step. A better guess significantly reduces the amount of re-work required to perform a complete fuel depletion calculation.

The program was developed first to determine the “burn requirement.” This requirement is the amount of reactivity that is lost in a lobe during the timestep to be analyzed by HELIOS. Assuming an energy release per fission of 200 MeV, an equivalent fuel burn in grams can be developed per megawatt day. The mass of fuel required to produce one megawatt for one day can be calculated using equation 2.

Equation 2

Where:

* is the fuel required in grams,
* is the molecular weight of U-235,
* is Avogadro’s number, and
* is the energy released per fission, taken to be 200 MeV/fission event.

The amount of uranium calculated by equation 2 is good for a clean core with no poison. Experimentally, this value has been adjusted for use in ATR due to poison effects within the fuel and reflector. For this algorithm and others that have relied on a similar calculation, is taken to be 1.3 grams of U-235 per MWd.

There are three different ways that fuel depletion is offset in the ATR. The first of these is the regulating rod which operates in automatic control and changes position to keep the reactor critical. As fuel is depleted, the regulating rod withdraws thereby adding positive reactivity. At a certain withdrawal position, operations personnel will with draw the outer shim control cylinders which also adds positive reactivity. This action causes the regulating rod to insert to offset the effect of the outer shim control cylinders. This process continues with the regulating rod moving out and then being driven back in with the outer shim control cylinders until the outer shim control cylinders are withdrawn to a certain point. There is no set rule for when the outer shims have been withdrawn too far, but as a general best practice, it is best to maintain the outer shim control cylinders between 80 and 120 degrees withdrawn. This allows the most reactive portion of the drum to be in the core at all times which is advantageous for reactivity control. When the outer shim control cylinders in a quadrant approach 120 degrees of rotation, a neck shim is withdrawn. This adds positive reactivity primarily to that quadrant which causes the operator to have to insert the outer shim control cylinders primarily in that quadrant.

Over the course of operating the reactor, it has been observed that neck shims that are more toward the periphery of the core tend to influence their quadrant more than those that are toward the center of the core. Those neck shims toward the center have a larger impact on the core in general with the reactivity addition being more equally divided among the lobes. In order to account for this division of neck shim reactivity, an additional algorithm was developed as part of the burner algorithm. This was done primarily by numeric analysis rather than by any method developed from first principles.

Each neck shim was assigned an effectiveness factor. This factor accounts for the division of quadrant neck shims to lobe locations. For example, 0.20$ from a neck shim isn’t divided into four quadrants, but into five lobes. Neck shim closer to the center provide more reactivity to the center lobe than those toward the outside of the core so they are assigned a lower effectiveness factor. Said differently, less of the neck shim reactivity is available to be assigned to each quadrant. For inner neck shims, positions 1 through 3, an effectiveness factor of 0.7 was assigned. For outer neck shims, positions 4 through 6, an effectiveness factor of 0.85 was assigned. Additional work could be done to determine an incremental effectiveness factor for each position, but for the scope of this project and considering the uncertainties inherent in the algorithm, these two divisions are likely sufficient.

Once an effectiveness has been assigned, the remaining worth is portioned among the lobes. As would be expected, those neck shims toward the center are more evenly distributed across the core while those toward the periphery are much more skewed to their respective quadrants. Table 1 shows the partitioning of the neck shims.

Table : Neck Shim Partitioning

|  |  |  |  |
| --- | --- | --- | --- |
| Neck Shim | % to quad | % to adjacent | % to opposite |
| 1 | 45% | 42% | 13% |
| 2 | 50% | 38% | 12% |
| 3 | 55% | 34% | 11% |
| 4 | 60% | 30% | 10% |
| 5 | 65% | 26% | 9% |
| 6 | 70% | 22% | 8% |

The percentage to quad column shows how much of the neck shim reactivity gets assigned to that quadrant. The next column, percent to adjacent shows how much of the reactivity get assigned to both adjacent quadrants. For example, the number 1 Northwest neck shim rod has 42% of its reactivity assigned to the Southwest and Northeast quadrants, or 21% assigned to each for a total of 42%. This is done for each neck shim in the algorithm. The reactivity that is assigned to each quadrant is added to the burn requirement for that quadrant for each time step. This, in effect, lessens the burn requirement for that time step.

As mentioned above there are a number of unquantifiable uncertainties within the core that this algorithm simply cannot account for. These uncertainties include things such as reflector poisoning, fuel element loading, fuel element poisons, fission product inventory from recycled fuel elements, and changes in experiment loading. Therefore, this algorithm is merely a first order gross approximation of what is actually happening in the core and is only meant to be a best first guess for the shim positions to be entered into HELIOS. HELIOS has the capability of accounting for and calculating all of the uncertainties mentioned above. As such, once the first best guess is entered HELIOS is run to determine what the actual deviation from criticality is. This is entered into the workbook and the algorithm is adjusted using a term defined as a “lumped fission product” which is essentially a bias to the algorithm to ensure that it matches the HELIOS predicted deviation from criticality. This correction factor aligns the algorithm with HELIOS effectively taking into account all of the uncertainties that exist in the calculation. This lumped fission product is divided into each quadrant based on the percentage of total power that quadrant represents. This process assumes that the reflector is depleted somewhat evenly, that the fuel is a mix of new and recycled in each quadrant, and that the experiment loading is somewhat similar to past cycles. The lumped fission product reactivity is then added to the total reactivity burden for each quadrant and the neck shim and outer shim positions can be determined anew using the new information. This second guess is often the last guess that needs to be made in order to get a HELIOS depletion run that is satisfactory. If the process is followed again, the third iteration will yield outer shim control cylinder positions and neck shim positions that are satisfactory.

One main caveat with the entire algorithm is that it is heavily numerically based. A basic assumption of a numerical based or observed set of rules is that if reactor operation differs significantly from what has historically been true, the algorithm may fail. Changes in reflector life, i.e., a clean core verses a reflector at end of life will change the build up of reflector poisons which will have an effect on the burn requirement. Perhaps using 1.3 g/MWd of U-235 will be excessive and should be changed to a lower value. Similarly, if the fuel loading is significantly changed the algorithm might have to be adjusted. Using quadrants or lobes with significantly depleted elements or completely fresh elements may have detrimental effects to the algorithm. However, the algorithm as currently build does converge very quickly and using the HELIOS predicted deviation from criticality as an input bias should serve to minimize these effects.

This algorithm can in no way be considered safety significant or important to safety and should not be used in place of HELIOS. It is a tool to provide a guess for HELIOS input and is intended to help the analyst make the best guess possible. The onus is still on the analyst to correctly identify shim positions for HELIOS to ensure that the cycle length can be met at the requested powers and that none of the fuel elements exceed any limits during the depletion calculation.

## Usage

A basic spreadsheet has been built that implements the burner algorithm and is used during phase 5 for the core physics analysis. Phase 5 is the depletion calculation for the cycle. In broad terms, the spreadsheet is used prior to performing any of the depletion calculations. It is used to get a best first-guess for the outer shim control cylinder positions and the time steps at which each neck shim should be withdrawn to compensate for fuel depletion. It is broken up into a series of steps that are outlined within the workbook. The steps will also be discussed here.

### Steps

The first step in utilizing the burner algorithm is entering the requested power division. The power division is based on experiment needs and can be found in the reactor loading record. The power division should be approximately the same as that used in phase 4 of the core physics analysis in which the outer shim control cylinder position was found to achieve the desired power split. These lobe powers are entered in megawatts and should be the nominal power requested. There are occasions when the analyst might choose to use a higher power in the first step than what was used during phase 4, specifically, if the tolerance around the nominal power division is very high, the analyst may want to use a higher power than nominal. This will help to ensure that enough reactivity is planned for the cycle. There is a utility in the algorithm to enter two different power divisions. This is used for cycles in which a portion of the cycle is operated with one power division and a portion of the cycle at another power division. This is often done during “soak” cycles. A soak cycle is run with a very flat power division for several days to soak a test or basically re-condition the test prior to the main testing. The main test is performed at very high power with a very large power division, primarily in the south. The experiments are driven into the core at high rates which induces an extreme temperature rise in the experiment thus simulating an accident scenario. These cycles are rare so normally, only the first power division will be used.

The second step is to input the reactivity worth of each neck shim as output by HELIOS in phase three of the core physics analysis. The algorithm uses the neck shim worth to determine how far to move each of the outer shim control cylinders in each quadrant to maintain a constant power. This step is important as the neck shim worth will change from beginning of life to end of life. On top of that, the neck shim worth changes as a function of local neutron flux which means that cycles with a large power tilt will have very different neck shim worth from cycles with a more moderate power tilt. This neck shim reactivity is partitioned to the different quadrants as described in the development section.

Step three is to enter the outer shim control cylinder position determined in phase three of the core physics analysis to achieve the required power split. These shim positions are the beginning for the burner algorithm and are the basis for ensuring that the power division is maintained. The entire algorithm is based on tracking and accounting for differences in reactivity in each quadrant. Thus, if the starting point is correct, i.e., the starting power division is right, the algorithm will be able to maintain that power division throughout all of the time steps.

Step four is where the analyst determines what burn steps should be used to get the desired result. The analyst can also specify which power division the algorithm should use to calculate the reactivity burden. The time steps are usually entered in full day increments, though the algorithm will handle partial day increments without any issue.

Once the four steps have been completed, the algorithm has enough information for the analyst to begin adjusting neck shim position to account for fuel depletion. The algorithm automatically positions the outer shim control cylinders to maintain power in their respective quadrants. Once a neck shim is withdrawn from the list of available neck shims, the reactivity is added to the quadrants for that time step and the outer shim control cylinders adjust automatically to compensate for the neck shim withdrawal. Once selected, the neck shim is automatically removed from the list of available neck shims which helps the analyst be sure that neck shims are only accounted for one time per cycle to match reality.

The resulting best guess is presented on a tab called HELIOS which has the outer shim control cylinder positions and neck shim positions for each time step. The outer shim control cylinder positions are conditioned for input into HELIOS, meaning they have been rounded to the nearest 3.75 degree. This is the requirement for HELIOS input. The neck shim positions, or rather the state in which the neck shims are withdrawn is left the same as the raw data. These positions are input into HELIOS and a full depletion calculation is performed.

Once the results of the depletion are performed, the resulting HELIOS values are input into the spreadsheet. First is the “HELIOS offset,” or the HELIOS predicted deviation from criticality for the startup timestep. This provides a baseline for the burner algorithm to begin to incorporate the HELIOS data. The beginning of step (BOS) deviation from critical is input into the algorithm. The workbook then calculates what the bias or lumped-fission product factor needs to be to ensure that the algorithm is providing the same results as HELIOS. This has to be done sequentially as each step is affected by the preceding step lumped fission product term. Once the lumped fission product terms have been added, the neck shims are re-selected based on the new outer shim control cylinder positions. This second iteration provides HELIOS input that is more likely than not an acceptable shim configuration for a full depletion calculation. If not, one more iteration is performed. For all of the testing that has been done, there has never been a cycle that required more than three iterations to determine acceptable shim positions for the cycle.

### Best Practices

The selection of neck shim withdrawal times is entirely up to the analyst. Presented here are some best practices that will help ensure success when using the algorithm. In the first iteration, the analyst should attempt to keep the deviation from critical less than 0.50$ in either direction, either super or sub-critical. This helps to ensure that the algorithm is conservative. Additionally, the variance between steps should be minimized. For example, if there is a deviation of 0.35$ from critical in the positive direction, the next time step should be kept around this value. It would not be a good idea for the next step to have a 0.35$ deviation from critical in the negative direction. This may cause the algorithm not to converge as quickly on a solution.

Neck shims should be withdrawn in the quadrant with the most fully withdrawn outer shims first. This is in an attempt to equalize the shim positions which helps the nuclear instruments in the reactor. The how and why of that particular phenomenon is best left for another project. Unless required to maintain the deviation from critical as mentioned above, neck shims should not be pulled unless the outer shims in a quadrant is approximately 110 degrees withdrawn. Care should be taken to try to minimize the difference between the most fully withdrawn set of outer shims and the most fully inserted pair of outer shims. Neck shims should be withdrawn starting from position six and working toward position one. There must be a specific documented reason to deviate from this pattern.

Finally, neck shim rods shouldn’t be withdrawn if they will cause any quadrant shims to dip below 80 degrees of rotation. This isn’t as important as those points outlined above but should also be considered. The reason for this is there are specific approvals that need to be obtained by operations prior to performing this type of evolution in the plant. There are some instances when a specific test will issue a requirement not to allow outer shims to be withdrawn more than 80 degrees and subsequently inserted below 80 degrees.

## Results

The specific results of the algorithm are unimportant as they change from cycle to cycle. The important thing is the reduction in time required to develop shim positions for a depletion run. As discussed in the background section of this algorithm, the program can reduce the time required to complete a phase five core physics analysis from weeks to days and is not dependent on the experience level of the performer. This time saving from this algorithm can be applied to other aspects of the core physics analysis which in the end will help reactor engineering personnel to deliver a higher quality product.

For every cycle in which this algorithm has been used to develop a best first guess, the utility and time savings has been apparent. There was one cycle that has been analyzed so far that required three iterations of the program. This was cycle 167A-1 which was a high power cycle with a very extreme power split. The algorithm maintained the required power correctly in all lobes for the requested time, but the critical state was more elusive. It is unclear whether the error was in the program, as it was still being developed at the time, or if there was an error made by the analyst. Either way, the algorithm converged on an adequate solution in three iterations. It is expected that for future cycles, only two iterations will be required.

# Tappendorf Validation

## Background

In 1988, Tappendorf who was at the time a reactor engineer for the Advanced Test Reactor conducted a series of tests using ATRC which is the ATR critical facility, a one-to-one mockup of the Advanced Test Reactor, to determine how the outer shim control cylinder worth changed as a function of relative power. This was done by measuring the reactivity change of the outer shim control cylinders for different relative power distributions. What he found was that a relationship existed in the form of equation 3 [1].

Equation 3

Where:

* is the reactivity of the shims at relative power , and
* is the reactivity of the shims at relative power .

Relative power in this application means that the given or calculated lobe power is normalized to 250 MW. This is done per equation 4.

Equation 4

Where:

* is the relative lobe power of interest,
* is the absolute power of the lobe of interest, either calculated or measured, and
* is the total absolute core power

This relationship was then utilized by reactor engineering personnel for many years and became a main reactivity calculation tool in a number of software applications. One of the main goals of this project was to determine if the Tappendorf relationship as presented in Equation 3 is valid in many of the cases in which it is used. Over the years, the relationship has been applied to safety rod worth calculation, experiment worth determinations, and fission product poison characterization. This presents a problem. Generally speaking, a relationship such as this is only valid for the case in which it was developed, i.e., outer shim control cylinders. Another curious thing about this relationship is that it is not reversible. Once a new reactivity is calculated for two given relative powers, this value cannot be used in the equation to produce the original value. This lack of reversibility was a major driver to perform a validation case using a qualified neutronic software.

## Methodology

While the reactivity worth of the fuel and boron were being determined, data was taken from each of those tests described in the Reactivity Determination section for relative power. The relative power change was calculated using a variant of the Tappendorf equation for the element in the test sequence for each testing position. This calculated power was compared to the HELIOS predicted relative power. This was one method of determining how well the Tappendorf equation matched the qualified software.

The second method utilized in Tappendorf validation was to use the HELIOS predicted relative power and the HELIOS predicted reactivity change of the fuel element in position 1 to predict what the fuel element reactivity change would be in each of the remaining 39 fuel element positions. This was done for every element in every position. This reactivity change was then compared against the reactivity change of the element as predicted by HELIOS. Results are as follows.

## Results

### 100 gram case

Figure 6 shows the result of the relative power prediction both using the Tappendorf equation and using HELIOS.

Figure : Relative Power Comparison

In figure 6, PB is the power before the perturbation. In this case it is the relative power of each lobe with all fresh fuel elements in the core. PF is the Tappendorf predicted relative power based on the fuel element with 100 grams less fuel being applied to every position. The model PF is the HELIOS predicted value. For almost every position there is very good agreement between the Tappendorf predicted relative power division and the HELIOS relative power division. There is a slightly larger deviation in the elements immediately before the center lobe elements and immediately after. These elements are known as the 2’s and 9’ positions and represent the locations in the core where the fuel transitions from concave in to concave out. This is likely an artifact of the way HELIOS has to differentiate relative power among the lobes rather than allowing a smooth power gradient among the fuel elements.

Wondering if the Tappendorf equation is sensitive to the magnitude of reactivity change, each element reactivity was plotted against the percent deviation from the HELIOS predicted relative power. This is shown in figure 7.

Figure : Percent Error as a function or reactivity change

The first thing that is immediately obvious in figure 7 is that there appears to be two separate data sets and that each data set appears to be linear. As the magnitude of the reactivity change increases, so does the percent deviation. Thus, elements toward the center of the core (higher reactivity) tend to produce slightly more deviation from the HELIOS predicted relative power. The other data set with the elements that have the largest reactivity change is comprised of the elements that make up the center lobe. This was a surprise but is evident in all test cases. These central elements when analyzed by Tappendorf produce error rates that are approximately 0.10%, far lower than the majority of the other elements. The reason for this is currently unknown. It could be speculated that the reason for this phenomenon is center power as calculated by HELIOS is a primarily a function of the power in the remaining four lobes. For elements in the center lobe, the outer lobe relative power division is mostly unchanged. Thus, the center power as calculated by HELIOS is relatively unchanged. That would account for the slope of the linear fit through these data points. The slope is slightly negative, but only slightly. Therefore, each element regardless of reactivity worth produces approximately the same result in the center lobe. Additional work would have to be done to validate this supposition or disprove it entirely. This work might consist of measured data rather than just calculated data.

The second test to determine if the Tappendorf method is valid was to use the HELIOS predicted change in relative power to determine what the fuel element would be worth in a different position. For example, the Tappendorf worth of fuel element two was calculated based on the HELIOS predicted worth of fuel element one for the HELIOS predicted power division. This was done for all elements in each position, meaning that element 1 was used to predict the other 39 elements and then element 2 was used to predict the other 39 elements, then element 3, and so on. Figure 8 shows the result of this comparison.



Figure : Reactivity Calculation Comparison

Due to the size of the dataset, it is difficult to see the entire figure in detail. Thus, it was colored to show the general trend. Cells that have been assigned green are those in which the percentage difference between the Tappendorf predicted worth and the HELIOS calculated worth is less than 25%. Cells without a color have an error between 25% and 50%. Anything in red is greater than 50% different. When colored this way a pattern emerges. The general pattern is that elements that are in the center of the core tend to predict other centrally located elements fairly well but do a terrible job at predicting the worth of elements on the outside. For example, element 1 reactivity may be used to predict the reactivity of elements 2-3, 8-13,18-23,28-33, and 38-40, but can’t do anything that ends in 4,5,6,or 7. However, elements that are on the outside of the core predict other elements on the outside of the core but not those in the center. And, it doesn’t matter what the change in relative power is, the results are the same as will be seen in the other cases. Figure 9 has been included as a “zoomed-in” picture to show more detail in the calculation.



Figure : Zoomed-in Reactivity Calculation Comparison

### 254 gram case

Figures 10 and 11 show the same data as figures 6 and 7 but for the 254-gram case. The plots are essentially the same, but have larger magnitudes.

Figure : Relative Power Comparison

Figure : Percent Error as a function or reactivity change

Figure 9 again shows the existence of two data-sets and is even more linearly correlated than that shown in the 100-gram case. Note that the maximum error for the 100-gram case was <1.2% but in this case it is approximately double. That is reasonable given the linear nature of the error. The error is approximately doubled because the reactivity change is approximately doubled.

The reactivity calculation comparison is shown in figure 12.



Figure : Reactivity Calculation Comparison

The same pattern from before is shown. Of particular interest here is the fact that there are less green cells than before and more un-colored cells. The larger the reactivity difference is, the worse the prediction becomes for elements that aren’t in the same locations.

### 300 gram case

Figures 13 and 14 present the same information for the 300 gram borated case. The non-borated case is not presented here because of similarities in the borated case. The test results are nearly identical.

Figure : Relative Power Comparison

Figure : Percent Error as a function or reactivity change

Figure 15 shows the reactivity calculation comparison for the 300-gram case.



Figure : Reactivity Calculation Comparison

Again the same pattern exists with slightly greater errors than in the 254-gram case. To better illustrate this point, table 2 is provided which shows the comparison in average reactivity difference among the tree cases presented here.

Table : Reactivity Comparison Differences



## Conclusion

The riddle of the Tappendorf equation being acceptable is solved. The conclusions reveal that the Tappendorf equation can be used, however there is a caveat. To correctly use the correlation, the reactivity worth to be calculated should be in the same or similar position to the position in which it was either measured or calculated. Using the Tappendorf equation to determine the worth change of an experiment from one side of the core to another is not appropriate. The worth change for an experiment that was measured in the NW flux trap can be determined for any relative power of the NW flux trap. Thus, the Tappendorf equation is particularly useful in determining safety rod worth, outer shim control cylinder worth, and neck shim rod worth. It is marginally useful in determining experiment worth changes for those experiments that are to be irradiated for multiple cycles in the same position. The correlation should not be used for experiments that are moved to different positions in the core.

These results beg the question, why? Why is the Tappendorf equation so positionally dependent? The answer lies in perturbation theory. The basis behind perturbation theory is that a very small perturbation in the flux or the flux shape produces a very small perturbation in the corresponding adjoint flux or flux shape. Thus the adjoint operator can be approximated by the original flux or flux shape and the terms then cancel out. This means that for small reactivity changes, the diffusion operator is self-adjoint, and we know that the perturbed flux can be approximated by the original flux [2]. When Tappendorf performed his experimentation, he measured the change in outer shim control cylinder for different relative powers. Because the flux shape is always the same for a given pair of outer shims, the resulting relationship became flux dependent. Of more importance however is the shape of the flux rather than the magnitude of the flux. Perhaps shape is the wrong word. It might be more correct to say the relative flux. This is the flux at the outer shim control cylinders relative to the rest of the core. As long as this relative flux is the same for any relative power, the diffusion operator is self-adjoint so the flux terms in the equations can be cancelled. In effect, Tappendorf derived a relationship to calculate perturbation theory for the Advanced Test Reactor purely by experimentation. This explains almost all of the phenomena seen throughout this project. It explains why for larger reactivity differences, there are larger errors. It explains why the error in calculated reactivity is linearly dependent on the magnitude of the reactivity change. It explains why a given element can be used to determine the worth of the same element in a different position with similar relative flux values but not other positions.

# Perturbation Theory

Perturbation theory can be applied to the Advanced Test Reactor in another way. For the test cases already reported, the reactivity change for a given amount of fuel can be predicted by HELIOS, and again by using Tappendorf’s equation. Running HELIOS is expensive from a computation and time standpoint, and the Tappendorf equation has limitations such that changes in relative flux cannot be accommodated. However, if perturbation theory is applied in a general high level sense to the six factor formula, the same results can be obtained.

The six factor formula is an expression to determine the effective multiplication factor, or a measure of the difference between neutrons produced and lost from one generation to the next. It is also known as the neutron life cycle. The six factor formula is presented in equation 4.

Equation 4

Where:

* is the effective multiplication factor, meaning the ratio of neutrons in the current generation to neutrons in the preceding generation,
* is the fast fission factor, a measure of fission produced by fast neutrons in U-238,
* is the fast non-leakage parameter or how many neutrons do not leave the system while at elevated energies,
* is the resonance escape probability or the chance a neutron has to slow to thermal energies without being parasitically captured by a U-238 nucleus,
* is the thermal non-leakage parameter meaning the fraction of thermal neutrons that do not leave the system,
* is the thermal utilization factor which is a measure of the ratio of neutrons absorbed in the fuel to everything else, and
* is the reproduction factor, how many neutrons are created for each fission event.

Approximations and guesses for each of these factors can be made for ATR. The fuel in the reactor is highly enriched so the fast fission factor is only slightly greater than one. For this exercise, it is assigned a value of 1.01. The fast and thermal non-leakage terms vary greatly depending on the position of the outer shim control cylinders, but have been assumed to be 0.799. The resonance escape probability is assumed to be 0.99 due to the limited number of U-238 atoms. The thermal reproduction factor is given as 2.02 for U-235.

Assuming a reactor that is perfectly critical meaning the effective multiplication factor is 1.0, the value for thermal utilization can be determined by re-arranging equation 4. Thus, the thermal utilization factor is determined to be 0.776. This is slightly lower than that given by the NRC [3]. This is probably due to differences in the ATR and the production reactors that are represented in the NRC documentation. However, for the benchmarking case with HELIOS, the eigen values collected were for outer shim control cylinders at 3.75 degrees which means additional hafnium is inserted into the core. Regardless, the numbers are close enough for demonstration purposes.

The thermal utilization factor can be defined per equation 5.

Equation 5

Where:

* is the macroscopic cross section of all isotopes of fuel, and
* is the macroscopic cross section of everything else in the core.

By assuming all new fuel in the reactor the U-235 inventory can be determined. The macroscopic cross section of everything else can be determined by back-solving equation 5. Using perturbation theory, the macroscopic cross section of everything else is assumed to be constant for each perturbation of fuel for the cases as presented already. Thus, the only reactivity change can be assumed to come from the change in fuel for each test case. It is also assumed that the change in reactivity from each case is small enough to allow the use of perturbation theory.

The effective multiplication factor is re-calculated based on the change in fuel element inventory for the entire core. This new k-value or eigen value is compared to the original case where k = 1. Additionally, the eigen value was also determined using a method given in Duderstadt [2] which assumes a single energy group, bare, un-relfected, cylindrical core. This approximation isn’t great given the complex reflected geometry of the ATR but is included here just for comparison. See the derivation given in Duderstadt for more information. The results are presented in table 3.

Table : Comparison of Reactivity Calculations with Perturbation Theory

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Reactivity Calculation for various methods | | | | |
|  | HELIOS | | Six Factor Formula | Duderstadt |
|  | Average | Center Elements Only |
| 100 | $ (0.07) | $ (0.12) | $ (0.07) | $ (0.15) |
| 254 | $ (0.16) | $ (0.29) | $ (0.18) | $ (0.38) |
| 300 | $ (0.17) | $ (0.30) | $ (0.21) | $ (0.45) |

Different choices of non-leakage term will result in different reactivity values as calculated by the six factor formula. There have been no studies that have been explicitly performed to know for sure what a reasonable value for each of the factors is for the ATR. However, this process can be used to get a good approximation of how the HELIOS model might change for a given change in reactivity. For example, is a core physics analysis has been completed and the cycle eigen value is known, using the six factor formula with the demonstrated application of perturbation theory will result in a new eigen value that is very close to what HELIOS might produce. This claim will of course have to be validated in the future, but the groundwork and process methodology are here. If successful, this will reduce the HELIOS computation time by allowing reactor engineering personnel to approximate the change in reactivity without having to run HELIOS.

# Conclusions/Recommendations

The objectives in this project have been completed. The reactivity worth for U-235 and B-10 in the ATR have been characterized. The values reported here should be updated and become the new thumb rules for use in fuel element selection. This will allow reactor engineering personnel to fully optimize fuel element selection and increase fuel economy while ensuring the cycle length can be met. The fuel element reactivity was calculated to be 2489 g U-235/$ reactivity. Boron was calculated by be worth 4.86 g B-10/$ reactivity.

A fuel depletion optimization algorithm has been developed that will reduce the time required to perform a full set of depletion calculations from 2 weeks to 2 days. This is accomplished by using the fuel reactivity worth as calculated in this project and accounting for the fuel lost during operation, the neck shim compensation for fuel depletion and a lumped fission product term to ensure that the algorithm produces results similar to those of HELIOS for any given cycle. A step by step usage case is presented in this work with recommendations for best practices to maximize success rates by inexperienced users.

The Tappendorf methodology has been validated with exceptions. While the correlation may be freely used to determine the reactivity change of an experiment, safety rod, or fuel element in the same position, it may not be used to determine the reactivity change of the same for a change in position. This was done by comparing the Tappendorf predicted relative power to the HELIOS calculated relative power and then by using the relative power at two states given by HELIOS to determine the reactivity worth of a fuel element in all of the other positions. Fuel elements in the central positions tend to be acceptable predictors of fuel element worth in other central locations but fail to produce usable results in outer fuel elements. The reverse is also true, fuel towards the outside of the core will predict fuel worth for other outside positions but not fuel in the center of the reactor. This is likely due to changes in relative flux in those positions. Tappendorf inadvertently created an equation to express first order perturbation theory in the Advanced Test Reactor.

Perturbation theory can be successfully used to predict the change in critical eigen value for the reactor using multiple methods. The first is to simply use the Tappendorf relationship. The second is to apply perturbation theory to the six factor formula. Another method is to approximate the ATR as a one group bare un-reflected cylindrical core and use the method outlined by Duderstadt, though this has an uncertainty greater than 50% and should only be used in the event one of the above methods aren’t able to be used.

# References

1. Tappendorf’s equation
2. Duderstadt pg 223